BAULKHAM HILLS HIGH SCHOOL

YEAR 11

HSC ASSESSMENT TASK

December 2009

MATHEMATICS
EXTENSION 1

Time allowed - 50 minutes
Plus 5 minutes reading time

DIRECTIONS TO CANDIDATES

- Attempt ALL questions.
- Allocated marks are indicated for each question.
- All necessary working should be shown.
- Write your teacher's name and your name on the cover sheet provided.
- At the end of the exam, staple your answers in order behind the cover sheet provided.

Question 1

Find the Cartesian equation for the equation in parametric form $x = 2sin\theta, y = 3cos\theta$ **2**

Question 2

Find all real values of a for which $P(x) = ax^3 - 8x^2 - 9$ is divisible by $(x - a)$ **2**

Question 3

If α, β,and γ are the roots of $x^3 - 3x + 1 = 0$.

Find:

i) $\alpha + \beta + \gamma$ **1**

ii) $\alpha\beta\gamma$ **1**

iii) $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$ **2**

Question 4

The variable point $(3t, 2t^2)$ lies on the parabola. Find the cartesian equation for this parabola. **2**

Question 5

For $P(x) = x^3 - 3x^2 - 3x + 10$

i) Show that $x = 2$ is a root of $P(x)$ **1**

ii) What is the product of the other two roots? **2**

Question 6

i) Sketch the parabola whose parametric equations are **1**

$$x = t, y = \frac{1}{2}t^2$$

ii) Clearly mark and label the directrix and the focal point **2**

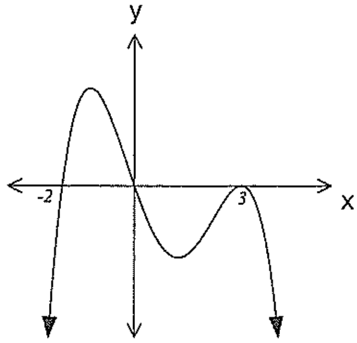
iii) Mark the point P and Q which correspond to $t = 1$ and $t = -2$ **2**

iv) Find the equation of the tangent at P and hence state the equation of the tangent at Q **3**

v) Show that the tangents intersect at R$(-\frac{1}{2}, -1)$ **2**

Question 7

Write down a possible equation of the polynomial with the following graph. **2**



Question 8

i) Express $F(x) = x^3 + 3x^2 - 10x - 24$ as a product of three linear factors **3**

ii) Sketch the curve **1**

iii) Hence solve, $x^3 + 3x - 10x - 24 > 0$ **1**

Question 9

- A monic polynomial $P(x)$ of degree 4 has zeros at 2 and -2 **5**
- $P(0) = 4$
- and $P(1) = -3$

Find $P(x)$ and hence solve $P(x) = 0$ for all real roots.

Question 10

A chord PQ of a parabola $x^2 = 4ay$ is such that it is always parallel to the line $y = x$, where P is $(2ap, ap^2)$ and Q is $(2aq, aq^2)$

i) Find the gradient of PQ and hence show that $p + q = 2$ **2**

ii) Show that the equation of the normal at P is $x + py = 2ap + ap^3$ **2**

iii) A normal is also drawn through Q. Find the point of intersection, R, of these two normals. **3**

iv) Find the locus of R. **2**

Assessment task Dec 2009. Ext 1. /44.

Quest. 1. (2 marks)

$\frac{x}{2} = \sin\theta \quad \frac{y}{3} = \cos\theta$

since

$\sin^2\theta + \cos^2\theta = 1$ 1

$\therefore \left(\frac{x}{2}\right)^2 + \left(\frac{y}{3}\right)^2 = 1$

$\frac{x^2}{4} + \frac{y^2}{9} = 1$ 1

Quest 2. (2 marks)

$P(x) = ax^3 - 8x^2 - 9$

$P(a) = a^4 - 8a^2 - 9 = 0$

$(a^2-9)(a^2+1) = 0$ 1

$a^2 = 9 \qquad a^2 = -1$

or 1 each answer $a = \pm 3 \leftarrow$ 1 $\searrow$ no soln

$\therefore \underline{a = \pm 3}$

Quest 3. (4 marks)

$x^3 - 3x + 1 = 0$

i) $\alpha+\beta+\gamma = \frac{-b}{a} = 0$ 1

ii) $\alpha\beta\gamma = \frac{-d}{a} = -1$ 1

iii) $\alpha\beta + \alpha\gamma + \beta\alpha = \frac{c}{a} = -3$ 1

$\therefore \frac{\alpha\beta+\alpha\gamma+\beta\alpha}{\alpha\beta\gamma} = \frac{-3}{-1}$

$= \underline{+3}$ 1

Quest 4. (2 marks)

let $x = 3t$, $y = 2t^2$ 1

$\frac{x}{3} = t \qquad y = 2\times\left(\frac{x}{3}\right)^2$

subin $\longrightarrow y = \frac{2x^2}{9}$ 1

or $x^2 = \frac{9}{2}y$

Quest 5. (3 marks)

$P(x) = x^3 - 3x^2 - 3x + 10$

i) $P(2) = (2)^3 - 3(2)^2 - 3\times2 + 10$ 1

$= 0$

$\therefore x = 2$ is a root of $P(x)$

ii) product of roots

$\alpha\beta\gamma = -\frac{d}{a}$ 1 method

$2\beta\gamma = -10$

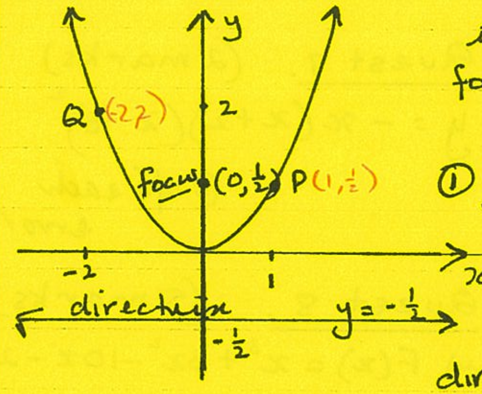
$\underline{\beta\gamma = -5}$ 1

Quest 6. (10 marks)

i) $x = t$, $y = \frac{1}{2}t^2$

$y = \frac{1}{2}x^2$ sketch 1

or $\underline{x^2 = 2y}$



ii) focus: $4a = 2$

$a = \frac{1}{2}$.

1 $\therefore (0, \frac{1}{2})$

1

directrix: $y = -a$

$y = -\frac{1}{2}$

iii) $t = 1 \quad P(2ap, ap^2) = (1, \frac{1}{2})$ 1

$t = -2 \quad Q(2aq, aq^2) = (-2, 2)$ 1

iv) $\underline{\text{tangent at } P}$. $(1, \frac{1}{2})$

$y = \frac{1}{2}x^2$

$\frac{dy}{dx} = x$ 1

at $x = 1 \quad m = 1$

$\therefore y - \frac{1}{2} = 1(x-1)$

$y = x - \frac{1}{2}$ 1

at $Q(-2, 2)$

$m = -2$

$\therefore\ y - 2 = -2(\cancel{x}+2)$

$y = -2\cancel{x} - 2$ ①

v.) $y = x - \frac{1}{2}$ --(i)

$y = -2x - 2$ --(ii)

equate ① + 2.

$x - \frac{1}{2} = -2x - 2$

$3x = -\frac{3}{2}$ ①

$x = -\frac{1}{2}$

sub into ①

$y = -\frac{1}{2} - \frac{1}{2}$ ①

$= -1$

$\therefore\ R\left(-\frac{1}{2}, -1\right)$

Quest 7. (2 marks)

$y = -x(x+2)(x-3)^2$

① off each error.

Quest 8. (5 marks)

i) $F(x) = x^3 + 3x^2 - 10x - 24$

try $1, -1, 2, -2$ etc.

$F(3) = 27 + 27 - 30 - 24$

$= 0$ ①

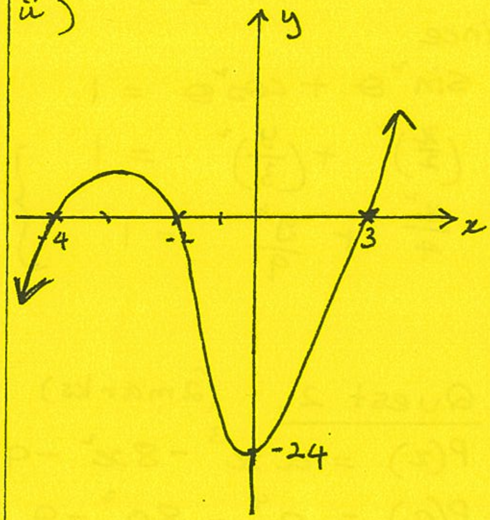
$(x-3)$ is a factor

$$\begin{array}{r|l} & x^2 + 6x + 8 \\ (x-3) & x^3 + 3x^2 - 10x - 24 \\ & x^3 - 3x^2 \\ \hline & \quad 6x^2 - 10x \\ & \quad 6x^2 - 18x \\ \hline & \qquad 8x - 24 \\ & \qquad 8x - 24 \\ \hline & \qquad 0 \end{array}$$ ①

$\therefore\ F(x) = (x-3)(x^2+6x+8)$

$= (x-3)(x+4)(x+2)$ ①

ii)



①

iii) $-4 < x < -2,\ x > 3.$ ①

Quest 9. (5 marks)

$P(x) = (x-2)(x+2)(x^2+bx+c)$ ①

$P(0) = (-2)(2)(c) = 4$

$-4c = 4$

$c = -1$ ①

$\therefore\ P(x) = (x-2)(x+2)(x^2+bx-1)$

$P(1) = (-1)(3)(1+b-1) = -3$

$-3b = -3$

$b = 1$ ①

$\therefore\ P(x) = (x-2)(x+2)(x^2+x-1)$

Solve $x^2 + x - 1 = 0$

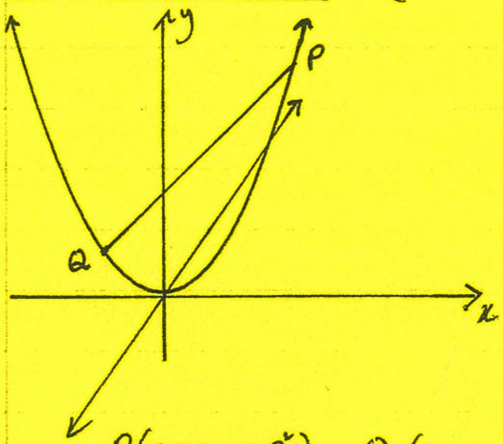
$x = \frac{-1 \pm \sqrt{1+4}}{2}$

$= \frac{-1 \pm \sqrt{5}}{2}$ ①

$\therefore$ Real roots of $P(x) = -2, 2, \frac{-1 \pm \sqrt{5}}{2}$

①

Question 10. (9 marks)



$P(2ap, ap^2)$ $Q(2aq, aq^2)$

i) $m_{PQ} = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{aq^2 - ap^2}{2aq - 2ap}$

$= \frac{\cancel{a}\cancel{(q-p)}(p+q)}{2\cancel{a}\cancel{(q-p)}}$

$= \frac{p+q}{2}$ ①

now for $y = x$

$m_1 = 1$

$\therefore m_{PQ} = m_1$

$\frac{p+q}{2} = 1$ ①

$p + q = 2$

ii) $x^2 = 4ay$

$y = \frac{x^2}{4a}$

$\frac{dy}{dx} = \frac{x}{2a}$

at $x = 2ap$ $m_1 = \frac{2ap}{2a}$

$= p.$

for normal $m_2 = -\frac{1}{m_1}$

$= -\frac{1}{p}$ ①

$y - ap^2 = -\frac{1}{p}(x - 2ap)$

$py - ap^3 = -x + 2ap$

$x + py = 2ap + ap^3$ ①

iii) at Q.

normal : $x + qy = 2aq + aq^3$ ①

R : $x = -py + 2ap + ap^3$ --(i)

$x = -qy + 2aq + aq^3$ --(ii)

subtract + substit.

$-py + 2ap + ap^3 = -qy + 2aq + aq^3$

$qy - py = 2aq - 2ap + aq^3 - ap^3$

$y(q-p) = 2a(q-p) + a(q-p) \times (q^2 + pq + p^2)$

① for working $y = 2a + a(p^2 + pq + q^2)$

$= a(p^2 + pq + q^2 + 2)$

sub in for x. --①

$x = -ap(p^2 + pq + q^2 + 2) + 2ap + ap^3$

$= \cancel{-ap^3} - ap^2q - apq^2 \cancel{- 2ap} \cancel{+ 2ap} \cancel{+ ap^3}$

$= -apq(p+q)$

①

$\therefore R: \left(-apq(p+q), a(p^2 + pq + q^2 + 2)\right)$ *

but $p + q = 2$ and $a\left((p+q)^2 - 2pq + pq + 2\right)$

either form OK here. *

$\therefore \left(-2apq, a(6 - pq)\right)$ *

Locus.

iv) $x = -2apq$ $y = a(6 - pq)$

$-\frac{x}{2a} = pq$

sub in ① working.

$y = a\left(6 + \frac{x}{2a}\right)$

$2y = 12a + x$

$x - 2y + 12a = 0$

$\therefore$ a straight line. ①